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The spontaneous generation of magnetic and chromomagnetic fields at high temperature in the standard model

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Abstract. The spontaneous generation of magnetic and chromomagnetic fields at high temperature is investigated in the standard model. The consistent effective potential including the one-loop and the daisy diagrams of all boson and fermion fields is calculated. The mixing of the generated fields due to the quark loop diagram is studied in detail. It is found that the quark contribution increases the magnetic and chromomagnetic field strengths as compared with the separate generation of fields. The magnetized vacuum state is stable due to the magnetic gauge field masses included in the daisy diagrams. Some applications of the results obtained are discussed.

1 Introduction

One of the interesting problems of present high energy physics is the generation of strong magnetic fields in the early universe. Different mechanisms of producing the fields at different stages of the universe evolution have been proposed (see, for instance, the surveys in [1–3]) and the influence of fields on various processes was discussed. In particular, the primordial magnetic fields, being implemented in the cosmic plasma, may serve as the seed source of the present extra-galaxy fields.

One of the mechanisms is a spontaneous vacuum magnetization at high temperature. This was investigated already for the case of pure $SU(2)$ gluodynamics in [4–6], where the possibility of this phenomenon has been shown. The stability of the magnetized vacuum was also studied [6]. As is well known, the magnetization takes place for the non-abelian gauge fields due to vacuum dynamics [7]. In fact, this is one of the distinguishable features of asymptotically free theories. In the papers mentioned the fermions were not taken into consideration. However, these may affect the vacuum state due to loop corrections in strong magnetic fields at high temperature.

In the present paper the spontaneous vacuum magnetization is investigated in the standard model (SM) of elementary particles. All boson and fermion fields are taken into consideration. In the SM there are two kind of non-abelian gauge fields – the $SU(2)$ weak isospin gauge fields responsible for weak interactions and the $SU(3)$ gluons mediating the strong interactions. The quarks possess both the electric and color charges, so they have to mix the chromomagnetic and the ordinary magnetic fields due to vacuum loops. Because of this mixing some specific configurations of the fields must be produced at high temperature. To elaborate this picture quantitatively, we calculate the effective potential (EP) including the one-loop and the daisy diagram contributions in the constant abelian chromomagnetic and magnetic fields, $H_c = \text{const}$ and $H = \text{const}$, at high temperatures.

Let us note the advantages of this approximation. The EP of the background abelian magnetic fields is the gauge fixing independent one. The daisy diagrams account for the most essential long-range correlation corrections at high temperature. Therefore, such a type of EP includes the leading and the next-to-leading terms in the coupling constants. Moreover, as was demonstrated in [6,8], the daisy diagrams of the charged gluons and the W-bosons make the spectra of these fields stable at high temperatures. This guarantees vacuum stability and the consistency of the approximation. The EP of this type has been used recently to investigate the electroweak phase transition in an external hypercharge magnetic field in the SM [9]. The obtained results are in good agreement with the nonperturbative calculations carried out in [10,11]. Therefore, it is reasonable to make use of the same approximation to investigate similar calculation procedure problems. A more detailed comparison of the results on the electroweak phase transition is given in [9]. A number of differences between the method in [10] and the present one having relevance to the problem under consideration will be discussed in the last section.

So we will use this approximation in what follows. Since an abelian magnetic hypercharge field is not generated spontaneously, in what follows we shall consider the non-abelian component of the magnetic field. The mechanisms of hypermagnetic field generation has been discussed in [8,12]. It will be shown that at high tempera-

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tures either the strong magnetic or the chromomagnetic fields are generated. They are stable in the approximation adopted due to the magnetic masses of $m^2_{\mathrm{transversal}} \sim (gH)^{1/2}T$ of the gauge field transversal modes [13]. In this way the consistent picture of the magnetized vacuum state in the SM at high temperature can be derived.

The contents of this paper are as follows. In Sect. 2 the contributions of bosons and fermions to the EP $v'(H,T)$ of external magnetic and chromomagnetic fields are calculated in a form convenient for numeric investigations. In Sect. 3 the field strengths are calculated. A discussion and concluding remarks are given in Sect. 4.

2 Basic formulae

The SM Lagrangian of the gauge boson sector is (see, for example, [14])

$$L = -\frac{1}{4}F^{\alpha}_{\mu\nu}F^{\mu\nu}_{\alpha} - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} - \frac{1}{4}\boldsymbol{F}^{\alpha}_{\mu\nu}\boldsymbol{F}^{\mu\nu}_{\alpha}, \qquad (1)$$

where the standard notation is introduced:

$$\begin{aligned} F^{\alpha}_{\mu\nu} &= \partial_\mu A^{\alpha}_{\nu} - \partial_\nu A^{\alpha}_{\mu} + g\epsilon^{abc}A^b_\mu A^c_\nu, \qquad (2)\\ G_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu,\\ \boldsymbol{F}^{\alpha}_{\mu\nu} &= \partial_\mu \boldsymbol{A}^{\alpha}_{\nu} - \partial_\nu \boldsymbol{A}^{\alpha}_{\mu} + g_s f^{abc}\boldsymbol{A}^b_\mu \boldsymbol{A}^c_\nu. \end{aligned}$$

The fields corresponding to the W-, Z-bosons and photons, respectively, are

$$\begin{aligned} W^{\pm}_{\mu} &= \frac{1}{\sqrt{2}}(A^1_\mu \pm \mathrm{i}A^2_\mu), \qquad (3)\\ Z_\mu &= \frac{1}{\sqrt{g^2+g'^2}}(gA^3_\mu - g'B_\mu),\\ A_\mu &= \frac{1}{\sqrt{g^2+g'^2}}(g'A^3_\mu + gB_\mu), \end{aligned}$$

and $\boldsymbol{A}^{\alpha}_{\mu}$ is the gluon field.

To introduce an interaction with the magnetic and chromomagnetic fields we replace all derivatives in the Lagrangian by the covariant ones,

$$\partial_\mu \to \mathcal{D}_\mu = \partial_\mu + \mathrm{i}g\frac{\tau^\alpha}{2}A^\alpha_\mu + \mathrm{i}g_s\frac{\lambda^\alpha}{2}\boldsymbol{A}^\alpha_\mu. \qquad (4)$$

Here τ^α and λ^α stand for the Pauli and the Gell-Mann matrices, respectively.

In the $SU(2)$ sector of the SM there is only one magnetic field, the third projection of the gauge field. In the $SU(3)_{\mathrm{c}}$ sector there are two possible chromomagnetic fields connected with the third and eighth generators of the $SU(3)$.

For simplicity, in what follows we shall consider the field associated with the third generator of the $SU(3)_{\mathrm{c}}$.

The introduction of an interaction with classical magnetic and chromomagnetic fields, as usual, is done by splitting the potentials in two parts:

$$\begin{aligned} A_\mu &= \bar{A}_\mu + A^{\mathrm{R}}_\mu, \qquad (5)\\ \boldsymbol{A}_\mu &= \bar{\boldsymbol{A}}_\mu + \boldsymbol{A}^{\mathrm{R}}_\mu, \end{aligned}$$

where A^{R} and $\boldsymbol{A}^{\mathrm{R}}$ describe the radiation fields and $\bar{A} = (0,0,Hx^1,0)$ and $\bar{\boldsymbol{A}} = (0,0,\boldsymbol{H}_3x^1,0)$ correspond to the constant magnetic and chromomagnetic fields directed along the third axes in the space and in the internal color and isospin spaces.

We used the general relativistic renormalizable gauge which is set by the following gauge fixing conditions [15]:

$$\begin{aligned} \partial_\mu W^{\pm\mu} \pm \mathrm{i}e\bar{A}_\mu W^{\pm\mu} \mp \mathrm{i}\frac{g\phi_{\mathrm{c}}}{2\xi}\phi^{\pm} &= C^{\pm}(x), \qquad (6)\\ \partial_\mu Z^\mu - \frac{\mathrm{i}}{\xi'}(g^2+g'^2)^{1/2}\phi_{\mathrm{c}}\phi_Z &= C^Z(x),\\ \partial_\mu \boldsymbol{A}^\mu + \mathrm{i}g_s\bar{\boldsymbol{A}} &= \boldsymbol{C}(x), \end{aligned}$$

where $e = g\sin\theta_{\mathrm{W}}$, $\tan\theta_{\mathrm{W}} = g'/g$, $\phi^{\pm}$ and ϕ_Z are the Goldstone fields, ξ and ξ' are the gauge fixing parameters, $C^{\pm}$ and C^Z are arbitrary functions and ϕ_{c} is the value of the scalar field condensate. Setting $\xi, \xi' = 0$ we choose the unitary gauge. In the restored phase the scalar field condensate $\phi_{\mathrm{c}} = 0$ and (6) are simplified.

The values of the macroscopic magnetic and chromomagnetic fields generated at high temperature will be calculated by minimization of the thermodynamic potential.

The thermodynamic potential Ω of the model is

$$\Omega = -\frac{1}{\beta}\log Z, \qquad (7)$$

$$Z = \mathrm{Tr}\exp(-\beta\mathcal{H}), \qquad (8)$$

where Z is the partition function, and $\mathcal{H}$ is the Hamiltonian of the system. The trace is calculated over all physical states.

To obtain the EP one has to rewrite (7) as a sum over quantum states calculated near the nontrivial classical solutions A^{ext} and $\boldsymbol{A}^{\mathrm{ext}}$. This procedure is well-described in the literature (see, for instance, [6,16,17]) and the result can be written in the form

$$\begin{aligned} V = V^{(1)}(H,\boldsymbol{H}_3,T) + V^{(2)}(H,\boldsymbol{H}_3,T) + ... \qquad (9)\\ + V_{\mathrm{daisy}}(H,\boldsymbol{H}_3,T) + ..., \end{aligned}$$

where $V^{(1)}$ is the one-loop EP; the other terms present the contributions of two-, three-, etc. loop corrections.

Among these terms there are ones responsible for dominant contributions of long distances at high temperature – the so-called daisy or ring diagrams (see, for example, [16]). This part of the EP, $V_{\mathrm{daisy}}(H,\boldsymbol{H}_3,T)$, is nonzero in the case when massless states appear in a system. The ring diagrams have to be calculated when the vacuum magnetization at finite temperature is investigated. In fact, one first must assume that the fields are nonzero, calculate the EP $V(H,\boldsymbol{H}_3,T)$ and after that check whether its minimum is located at nonzero H and $\boldsymbol{H}_3$. On the other hand, if one investigates problems in the applied external fields, the charged fields become massive with the masses depending on $\sim (gH)^{1/2}$, $\sim (g_s\boldsymbol{H}_3)^{1/2}$ and have to be omitted.

The one-loop contribution to EP is given by the expression

$$V^{(1)} = -\frac{1}{2}\mathrm{Tr}\log G^{ab}, \qquad (10)$$

where G^{ab} stands for the propagators of all quantum fields $W^{\pm}$, $\boldsymbol{A}$, ... in the background fields H and $\boldsymbol{H}_3$. In the proper time formalism, the s-representation, the calculation of the trace can be carried out in accordance with the formula [18]

$$\mathrm{Tr}\log G^{ab} = -\int_0^{\infty} \frac{\mathrm{d}s}{s} \mathrm{tr}\exp(-\mathrm{i}sG^{-1}_{ab}). \qquad (11)$$

Details of the calculations based on the s-representation and formula (12) can be found in [19–21].

We make use of the method of [19] allowing one in a natural way to incorporate the temperature into this formalism. A basic formula of [19] connecting the Matsubara Green functions with the Green functions at zero temperature is needed,

$$G^{ab}_k(x,x';T)=\sum_{-\infty}^{+\infty}(-1)^{(n+[x])\sigma_k} G^{ab}_k(x-[x]\beta u, x'-n\beta u), \qquad (12)$$

where G^{ab}_k is the corresponding function at $T=0$, $\beta = 1/T$, $u=(0,0,0,1)$, $[x]$ denotes the integer part of x_4/β, $\sigma_k = 1$ in the case of physical fermions and $\sigma_k = 0$ for boson and ghost fields. The Green functions in the right-hand side of (12) are the matrix elements of the operators G_k computed in the states $|x',a\rangle$ at $T=0$, and in the left-hand side the operators are averaged over the states with $T \neq 0$. The corresponding functional spaces U^0 and U^T are different but in the limit of $T \to 0$ U^T transforms into U^0.

The terms with $n=0$ in (12) and (10) give the zero temperature expressions for the Green functions and the effective potential V', respectively. So we can split it into two parts:

$$V'(H,\boldsymbol{H}_3,T) = V'(H,\boldsymbol{H}_3) + V'_\tau(H,\boldsymbol{H}_3,T). \qquad (13)$$

The standard procedure to account for the daisy diagrams is to substitute the tree level Matsubara Green functions in (10), $[G^{(0)}_i]^{-1}$, by the full propagator $G^{-1}_i = [G^{(0)}_i]^{-1} + \Pi(H,T)$ (see for details [6,16,17]), where the last term is the polarization operator at finite temperature in the field taken at zero longitudinal momentum $k_l = 0$.

Passing the detailed calculations we can notice that the exact one-loop EP will be transformed into EP, which contains the daisy diagrams as well as one-loop diagrams, by adding a term containing the temperature dependent mass of the particle to the exponent.

It is convenient for what follows to introduce the dimensionless quantities: $x = H/H_0$ ($H_0 = M_W^2/e$), $y = \boldsymbol{H}_3/\boldsymbol{H}^0_3$ ($\boldsymbol{H}^0_3 = M_W^2/g_s$), $B = \beta M_W$, $\tau = 1/B = T/M_W$, $v = V/H_0^2$.

The total EP in our consideration consists of several terms:

$$v' = \frac{x^2}{2} + \frac{y^2}{2} + v'_{\mathrm{leptons}} + v'_{\mathrm{quarks}} + v'_{W-\mathrm{bosons}} + v'_{\mathrm{gluons}}. \qquad (14)$$

These terms can be exactly written for the SM fields (in dimensionless variables).

(1) leptons:

$$v'_{\mathrm{leptons}} = -\frac{1}{4\pi^2}\sum_{n=1}^{\infty}(-1)^n \int_0^{\infty}\frac{\mathrm{d}s}{s^3} \cdot \mathrm{e}^{-(m^2_{\mathrm{leptons}}s + (\beta^2 n^2)/(4s))}(xs\mathrm{Coth}(xs) - 1); \qquad (15)$$

(2) quarks:

$$v'_{\mathrm{quarks}} = -\frac{1}{4\pi^2}\sum_{f=1}^{6}\sum_{n=1}^{\infty}(-1)^n \int_0^{\infty}\frac{\mathrm{d}s}{s^3}\mathrm{e}^{-(m_f^2 s + (\beta^2 n^2)/(4s))} \cdot (q_f xs\mathrm{Coth}(q_f xs)\cdot ys\mathrm{Coth}(ys) - 1); \qquad (16)$$

(3) W-bosons (see [22]):

$$v'_W = -\frac{x}{8\pi^2}\sum_{n=1}^{\infty}\int_0^{\infty}\frac{\mathrm{d}s}{s^2}e^{-(m_W^2 s + (\beta^2 n^2)/(4s))} \cdot \left[\frac{3}{\mathrm{Sinh}(xs)} + 4\mathrm{Sinh}(xs)\right]; \qquad (17)$$

(4) gluons (see [6]):

$$v'_{\mathrm{gluons}} = -\frac{y}{4\pi^2}\sum_{n=1}^{\infty}\int_0^{\infty}\frac{\mathrm{d}s}{s^2}\mathrm{e}^{-(m^2_{\mathrm{gluons}}s + (\beta^2 n^2)/(4s))} \cdot \left[\frac{1}{\mathrm{Sinh}(ys)} + 2\mathrm{Sinh}(ys)\right]. \qquad (18)$$

Here, m_{leptons}, m_f, m_W and m_{gluons} are the temperature masses of leptons, quarks, W-bosons and gluons, respectively; $q_f = (2/3, -1/3, -1/3, 2/3, -1/3, 2/3)$ are the charges of the quarks.

Since we investigate the dynamics of high-temperature effects connected with the presence of external fields, we used only the leading in temperature terms of the Debye masses of the particles ([6,22]).

The temperature masses of leptons and quarks are

$$m^2_{\mathrm{leptons}} = \left(\frac{e}{\beta}\right)^2, \quad m_f^2 = \left(\frac{e}{\beta}\right)^2. \qquad (19)$$

As is known [6], the transversal components of the charged gluons and W-bosons have no temperature masses of order $\sim g_s\boldsymbol{H}_3$ and $\sim gH$. Only the longitudinal components have Debye masses, but they are H- and $\boldsymbol{H}_3$-independent; therefore, they can be omitted in our consideration. Instead, the transversal component masses, which depend on the Landau level number, must be used. So the transversal temperature masses of W-bosons and charged gluons,

$$m_W^2 = 15\alpha_{\mathrm{e.w.}}\frac{h^{1/2}}{\beta}, \quad m^2_{\mathrm{gluons}} = 15\alpha_{\mathrm{s}}\frac{h^{1/2}}{\beta}, \qquad (20)$$

are to be substituted. Here, $\alpha_{\mathrm{e.w.}}$ and α_{s} are the electroweak- and the strong-interaction couplings, respectively.

In the approximation adopted in the present investigation we take as the masses the ground state energies of the transversal modes [13].

In the one-loop order the neutral gluon contribution is a trivial $\boldsymbol{H}_3$-independent constant which can be omitted. However, these fields are long-range states and they do give a $\boldsymbol{H}_3$-dependent EP through the correlation corrections depending on the temperature and field. We included only the longitudinal neutral modes because their Debye masses $\Pi^0(y,\beta)$ are nonzero. The corresponding EP is [6]

$$v_{\rm ring} = \frac{1}{24\beta^2}\Pi^0(y,\beta) - \frac{1}{12\pi\beta}\left(\Pi^0(y,\beta)\right)^{3/2} + \frac{\left(\Pi^0(y,\beta)\right)^2}{32\pi^2}\left[\log\left(\frac{4\pi}{\beta(\Pi^0(y,\beta))^{1/2}}\right) + \frac{3}{4} - \gamma\right]; \quad (21)$$

γ is Euler's constant, $\Pi^0(y,\beta) = \Pi^0_{00}(k=0,y,\beta)$ is the zero-zero component of the neutral gluon field polarization operator calculated in the external field at finite temperature and taken at zero momentum [6]

$$\Pi^0(y,\beta) = \frac{2g^2}{3\beta^2} - \frac{y^{1/2}}{\pi\beta} - \frac{y}{4\pi^2}. \quad (22)$$

Equations (14)–(18) and (21) will be used in the numeric calculations.

3 Generation of magnetic and chromomagnetic fields

In order to find the strengths of the generated magnetic and chromomagnetic fields we have to find the minima of the EP in the presence of both of them. First of all we will find the x and y strengths of the fields, when the quark contribution is divided in two parts:

$$v'_{\rm quarks}(x,\beta) = v'_{\rm quarks}|_{y\to 0} = -\frac{1}{4\pi^2}\sum_{f=1}^{6}\sum_{n=1}^{\infty}(-1)^n\int_0^\infty \frac{{\rm d}s}{s^3}e^{-(m_f^2 s + (\beta^2 n^2)/(4s))} \cdot(q_f xs{\rm Coth}(q_f xs) - 1) \quad (23)$$

and

$$v'_{\rm quarks}(y,\beta) = v'_{\rm quarks}|_{x\to 0} = -\frac{1}{4\pi^2}\sum_{f=1}^{6}\sum_{n=1}^{\infty}(-1)^n\int_0^\infty \frac{{\rm d}s}{s^3}e^{-(m_f^2 s + (\beta^2 n^2)/(4s))} \cdot(ys{\rm Coth}(ys) - 1), \quad (24)$$

where $v'_{\rm quarks}(x,\beta)$ is in the magnetic field, and $v'_{\rm quarks}(y,\beta)$ in the presence of the chromomagnetic field.

Let us rewrite the v' in (14) as follows:

$$v'(\bar{x},\bar{y}) = v_1(\bar{x}) + v_2(\bar{y}) + v_3(\bar{x},\bar{y}), \quad (25)$$

where $\bar{x} = x + \delta x$, $\bar{y} = y + \delta y$, and δx and δy are the field corrections connected with the effect of the fields' interfusion in the quark sector.

Since the mixing of fields due to a quark loop is weak (this will be justified by numeric calculations) we can assume that $\delta x \ll 1$ and $\delta y \ll 1$, and write

$$v_1(\bar{x}) = v_1(x+\delta x) = v_1(x) + \frac{\partial v_1(x)}{\partial x}\delta x,$$
$$v_2(\bar{y}) = v_2(y+\delta y) = v_2(y) + \frac{\partial v_2(y)}{\partial y}\delta y,$$
$$v_3(\bar{x},\bar{y}) = v_3(x+\delta x, y+\delta y) = v_3(x,y). \quad (26)$$

After simple transformations we can find δx and δy:

$$\delta x = \frac{\frac{\partial v_3(x,0)}{\partial x} - \frac{\partial v_3(x,y)}{\partial x}}{\frac{\partial^2 v_1(x)}{\partial x^2}},$$
$$\delta y = \frac{\frac{\partial v_3(0,y)}{\partial y} - \frac{\partial v_3(x,y)}{\partial y}}{\frac{\partial^2 v_2(y)}{\partial y^2}}. \quad (27)$$

Hence we may obtain $\bar{x} = x + \delta x$ and $\bar{y} = y + \delta y$.

The results on the field strengths determined by numeric investigation of the total EP are summarized in Tables 1 and 2.

In the first column of Tables 1 and 2 we show the inverse temperature. In the second one the strength of magnetic and chromomagnetic fields are adduced in the case of the quark EP, which describes each field separately. The next column gives the field corrections in the case of total quark EP. The fourth column presents the relative value of the corrections. The last column gives the resulting strength of magnetic and chromomagnetic fields, respectively.

As is seen, the increase of the inverse temperature leads to decreasing strengths of the generated fields. This dependence is well in accordance with the picture of the universe cooling.

From the above analysis it follows that at high temperatures the value of the each type of magnetic field is increased when the other one is taken into account. With temperature decreasing this effect becomes less pronounced and disappears at comparably low temperatures $\beta \sim 1$.

4 Discussion

Let us discuss the results obtained. As we elaborated in the approximation to the EP including the one-loop and the daisy diagrams, in the SM at high temperatures both the magnetic and chromomagnetic fields have to be generated. These states are stable, as follows from the absence of imaginary terms in the EP minima.

If the quark loops are discarded, both of the fields can be generated in the system separately. All these states are stable, due to the magnetic mass $\sim g^2(gH)^{1/2}T$ of the transversal gauge field modes. Here it worth to mention that the one-loop transversal gauge field mass is of order

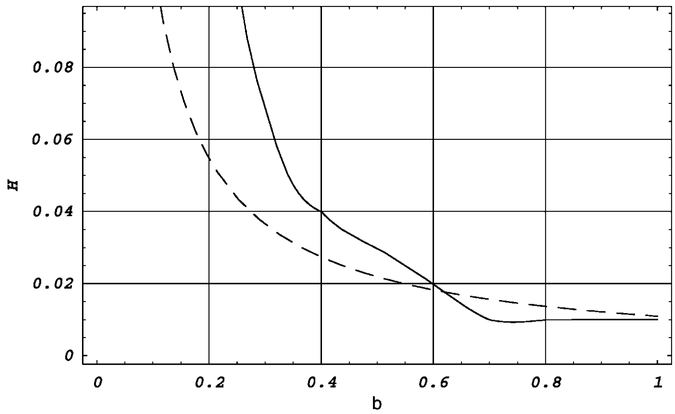

Fig. 1. The dependence of the strengths of the generated magnetic field (H) on inverse temperature (b). The *dashed line* is the theoretical position in the case of a single magnetic field and the *solid one* is calculated in the presence of both fields

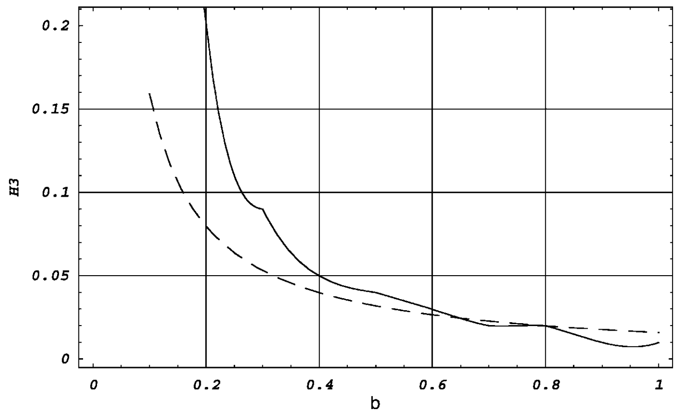

Fig. 2. The dependence of the strengths of the generated chromomagnetic field ($H3$) on inverse temperature (b). The *dashed line* is the theoretical position in the case of a single chromomagnetic field and the *solid one* is calculated in the presence of both fields

Table 1. The strength of the generated magnetic field

β	x	δx	$\delta x/x$,%	$\bar{x}$
0.1	0.7	0.0000165	0.002	0.7000165
0.2	0.2	0.000745	0.373	0.200745
0.3	0.07	−0.0000549	−0.079	0.0699451
0.4	0.04	−0.0000358	−0.090	0.0399642
0.5	0.03	−0.0000467	−0.156	0.0299533
0.6	0.02	−0.0000492	−0.246	0.0199508
0.7	0.01	−0.0000380	−0.380	0.0099620
0.8	0.01	−0.0000619	−0.619	0.0099381
0.9	0.01	−0.0000241	−0.241	0.0099759
1.0	0.01	−0.0000357	−0.357	0.0099643

Table 2. The strengths of the generated chromomagnetic field

β	y	δy	$\delta y/y$,%	$\bar{y}$
0.1	0.8	0.000301	0.038	0.800301
0.2	0.2	−0.000239	−0.119	0.199761
0.3	0.09	−0.0000988	−0.110	0.0899012
0.4	0.05	−0.0000884	−0.177	0.0499116
0.5	0.04	−0.000112	−0.280	0.039888
0.6	0.03	−0.0000982	−0.327	0.0299018
0.7	0.02	−0.0000442	−0.221	0.0199558
0.8	0.02	−0.0000733	−0.367	0.0199267
0.9	0.01	−0.000117	−1.166	0.009883
1.0	0.01	−0.000175	−1.749	0.009825

$\sim g^4T^2$, as the nonperturbative calculations predict. This estimate is found because the magnetic field strength of the spontaneously generated fields is of order $(gH)^{1/2} \sim g^2T$ [5,6]. The possibility to calculate the magnetic mass in perturbation theory is due to the approach when an external field is taken into consideration exactly when the polarization operator of the gauge field is calculated [13]. If one accounts for the magnetic field perturbatively, a zero value will be obtained [23].

The result on the stabilization of the spectra of charged gauge fields in the external fields at high temperature is very important. It has relevance not only for the problem of the consistent description of the generation of magnetic fields but also for the related problem of the symmetry behavior in external magnetic fields in the standard model investigated recently in [8,12]. In more detail the case of an external hypercharge magnetic field has been considered by both the perturbative [9] and nonperturbative [10] methods. In the latter paper, in particular, the unexpected result – the absence of the lattice structure condensate formed by the W-boson, Z-boson and electromagnetic fields – was obtained at high temperature for the values of the external hypermagnetic field corresponding to known estimates when the condensate has to appear. However, from the analysis carried out in the present paper it follows that the W-boson spectrum remains stable when the gauge field magnetic mass is included in the consideration. So, no causes for instability of the vacuum with the homogeneous magnetic field exist. The results of the impossibility of a strong first-order phase transition due to a strong hypermagnetic field obtained in perturbative [9] and nonperturbative [10] calculations are in agreement with each other. A more detailed comparison of the approaches and the results of nonperturbative and perturbative calculations in the external fields are given in [9].

As is seen from Figs. 1 and 2, presenting the results of numeric computations within the exact EP, the strengths of the generated fields are increasing with the temperature rising. It is also found that the curves obtained in a high-temperature expansion of the EP [6] are in good agreement with our numeric calculations.

The ground state possessing magnetic and chromomagnetic fields makes an advantage for the existence of these fields in the electroweak transition epoch. The state with the fields is stable in the whole considered temperature interval. The imaginary part in the EP exists for the external fields much stronger than the strengths of the spontaneously generated ones. The interfusion of magnetic and chromomagnetic fields arising from the quark sector of the EP is weak. The change of the field minima in the inclusion of the fields mixing does not exceed 2 per cents.

During the cooling of the universe the strengths of the generated fields are decreasing, which is in agreement with what is expected in cosmology.

One of the consequences on the results obtained is the presence of a strong chromomagnetic field in the early universe, in particular, at the electroweak phase transition and, probably, until the deconfinement temperature. The influence of this field on the transitions may bring new insight in these problems. As our estimate showed, the chromomagnetic field is as strong as the magnetic one. So the role of strong interactions in the early universe in the presence of the field needs more detailed investigations as compared to what is usually assumed [3].

We would like to finish with the remark that in the literature devoted to investigations of the quark–gluon plasma in the deconfinement phase carried out by non-perturbative methods, the vacuum magnetization at high temperature has not been accounted for (see, for instance, the recent paper [24] and references therein). From the point of view of the present analysis (as well as other studies carried out already in perturbation theory [4–6]) these investigations are incomplete. The generation of the chromomagnetic field at high temperature has to be taken into consideration. In this case a lot of discrepancies between the results obtained by different methods could be removed.

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